

# Quantum Effects in the Conductivity of a Quasi 2D Electron Gas

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We consider the role of the third dimension in the conductivity of a quasi 2D electron gas. If the transverse correlation radius of the scattering potential is smaller than the width of the channel, i.e. the width of the transverse electron density distribution, then scattering to higher levels of the confinement potential becomes important, which causes a broadening of the current flow profile. The resulting conductivity is larger than that obtained from a 2D Boltzmann equation. A magnetic field, parallel to the driving electric field, effectively competes with the confining potential and, in the limit of a strong magnetic field, it is the field, which largely shapes the electron and current profile, rather than the potential. As a result the current flow profile increases and a negative longitudinal magnetoresistivity of the quasi 2D electron gas may be observed.

## I. INTRODUCTION

Impurity-limited conductivity of a quasi-two-dimensional electron gas (quasi-2DEG) is usually calculated by means of a quasi-classical 2D Boltzmann equation (see, e.g. [1, 2, 3, 4, 5, 6]) and references therein). Quantum corrections to the 2D conductivity are assumed to be only due to the weak localization or interaction mechanisms. The starting point of the quasi-classical approach is the Hamiltonian

$$\begin{aligned} \hat{H} = & \sum_{\alpha, \mathbf{k}_2} \left( E_{\alpha} + \frac{\mathbf{k}_2^2}{2m} \right) a_{\alpha, \mathbf{k}_2}^{\dagger} a_{\alpha, \mathbf{k}_2} \\ & + \sum_{\alpha, \beta, \mathbf{k}_2, \mathbf{q}_2} M_{\alpha, \beta}(\mathbf{q}_2) e^{i\mathbf{q}_2 \mathbf{r}_2} a_{\alpha, \mathbf{k}_2 + \mathbf{q}_2/2}^{\dagger} a_{\beta, \mathbf{k}_2 - \mathbf{q}_2/2}, \end{aligned} \quad (1)$$

where

$$M_{\alpha, \beta}(\mathbf{q}_2) = \int dz \Psi_{\alpha}^{*}(z) U(\mathbf{q}_2, z) \Psi_{\beta}(z). \quad (2)$$

Here  $U(\mathbf{q}_2, z) = \int d^2r_2 e^{-i\mathbf{q}_2\mathbf{r}_2} U(\mathbf{r}_2, z)$  where  $U(\mathbf{r}_2, z)$  is a random scattering potential; the subscript 2 denotes here and below the 2D in-plane vectors.  $E_\alpha$  is the  $\alpha$ -th eigenenergy of the quantum well,  $V(z)$ , which confines the motion of the electrons in the  $z$  direction.  $\Psi_\alpha(z)$  is the corresponding eigenfunction.

In order to derive a quasi-classical Boltzmann equation one assumes that only the highest occupied level,  $\alpha_0$ , of the potential  $V(z)$  is of importance and all transitions, both real and virtual, to lower or higher levels can be discarded. If  $\lambda_z$  characterizes the width of the wave function  $\Psi_{\alpha_0}(z)$ , then the characteristic energy interval between the state  $E_{\alpha_0}$  and the neighboring states can be estimated as  $\Delta E_{\alpha_0} \approx \frac{\hbar^2}{2m\lambda_z^2}$ . Then the above assumption for the real transitions is justified if the channel is narrow enough and the temperature is low enough,  $\Delta E_{\alpha_0} \gg k_B T$ .

Virtual transitions to other states may be caused by the random scattering potential  $U(\mathbf{q}_2, z)$ . Its off-diagonal matrix elements (2) are negligible if the scattering potential is smooth, i.e. it is characterized by a large correlation radius,  $r_c \gg \lambda_z$ . Then the scattering is responsible only for an in-plane relaxation of the electron momentum of the non-equilibrium 2DEG. One may then discard any possible renormalization of the current due to an admixture of states with  $\alpha \neq \alpha_0$  and retain only the diagonal matrix elements  $M_{\alpha_0, \alpha_0}(\mathbf{q})$  in Eq. (1). Then a quasi-classical 2D Boltzmann equation follows straightforwardly. If one is interested in the profile of the current flow density along the  $z$  coordinate, it coincides in this case with the electron density profile, determined by the wave function of the  $\alpha_0$  level, i.e.,  $j(z) \sim \rho_{\alpha_0}(z) = |\psi_{\alpha_0}(z)|^2$ . The width of this profile may be called width of the channel.

A completely different situation takes place in the opposite limit,  $r_c \ll \lambda_z$ , to be called below the quantum limit. The scattering potential  $U(\mathbf{q}_2, z)$  induces strong transitions to other levels  $\alpha \neq \alpha_0$  of the quantum well. The matrix elements  $M_{\alpha, \beta}(\mathbf{q}_2)$  with  $\alpha, \beta \neq \alpha_0$  cannot be discarded, and the conventional approach based on a quasi-classical 2D Boltzmann equation is not applicable. An admixture of other states with the wave functions, localized in much wider regions than that of  $\psi_{\alpha_0}(z)$ , may lead to a much broader  $z$  profile of the current density. Its decay along the  $z$  axis is characterized by a length  $b$ , which we call effective width of the channel. It may essentially exceed the width  $\lambda_z$ .

The conductivity of a quasi-2DEG may be now sensitive to an external in-plane magnetic field, which may influence the  $\lambda_z$  value and lead to a corresponding increase of the effective

width of the channel, which carries the current flow. As a result, a negative magnetoresistivity in an in-plane magnetic field is expected. It is worthwhile to distinguish this mechanism of the longitudinal magnetoresistivity from the other mechanism recently proposed in Ref. [7], which considers a 2D system without any account of the third dimension. The magnetic field polarizes electron spins and causes a change of the Fermi energy and, hence, of the scattering time. It is emphasized that the mechanism, we propose here, does not consider electron spins or their polarization at all. It is of a crucial importance below that the system is *quasi two-dimensional* rather than really *two-dimensional*.

## II. 2D - CONDUCTIVITY

A calculation of the conductivity of a quasi-2DEG in the quantum limit  $\lambda_z \gg r_c$  cannot be carried out within the framework of a conventional quasiclassical 2D Boltzmann equation. A quantum approach is necessary, which takes into account the renormalization of all relevant quantities due to an admixture of various levels,  $\alpha \neq \alpha_0$ , of the quantum well. We shall see below that it is equivalent to explicitly accounting for the  $z$  dependence of the current density flow. This analysis can be best carried out using a quantum kinetic equation for the Wigner function. A detailed discussion of a gauge invariant derivation and analysis of such an equation is presented in our papers [8, 9, 10]. A discussion of quantum kinetic equations can be found in books [11, 12], which present also introductions to the diagrammatic technique, proposed originally by Keldysh [13].

We consider here a model, in which calculations can be carried out analytically. The confinement potential is harmonic,  $V(z) = \frac{m\omega^2 z^2}{2}$  with the frequency  $\omega$ ;  $m$  is the electron mass. We simplify the problem by assuming that only the ground state of the harmonic oscillator is occupied,  $\alpha_0 = 0$ , then  $\lambda_z = u \equiv \sqrt{\frac{\hbar}{m\omega}}$ ;  $u$  is the amplitude of the zero-point oscillations. We assume also that electrons can move freely in the  $xy$ -plane and are scattered by a potential, which is on the average homogeneous and isotropic in the whole three dimensional space. Its fluctuations are characterized by a certain correlation radius  $r_c$ . This radius can be neglected as compared to any scales important for the motion in the  $xy$  plane. However, following the above discussion we shall keep a finite value of  $r_c$  when considering the  $z$  motion of the electrons. Then

$$\langle U(\mathbf{r}_2, z)U(\mathbf{r}'_2, z') \rangle = \overline{U^2} \delta(\mathbf{r}_2 - \mathbf{r}'_2) l(z - z'). \quad (3)$$

Here  $\overline{U^2}$  is the mean square fluctuations of the scattering potential. If the scattering potential is created by short-range defects with a concentration  $c$ , then  $\overline{U^2} = cU_d^2$  where  $U_d$  is the scattering potential of a defect. The correlation function for the fluctuations in the  $z$  direction is chosen in the Gaussian form

$$l(z - z') = \frac{1}{r_c \sqrt{2\pi}} \exp[-(z - z')^2 / 2r_c^2]. \quad (4)$$

Calculating various space conditional moments of the quantum kinetic equation one obtains an infinite set of equations, providing the so called hydrodynamic formulation of the problem (for details see [10]). We restrict ourselves by the  $s$ - scattering in the  $xy$ -plane, which implies that the equations for the kinetic moment are decoupled from the equations for higher powers of the electron momentum. In order to receive these three equations we first take Eq. (31) of our paper [10], multiply it by the 3D electron momentum  $\mathbf{p}$ , and integrate its left and right hand parts over the variables  $\varepsilon$  and  $\mathbf{p}$ . The resulting equation for the  $z$  component of the nonequilibrium part of the conditional moment is trivial with zeros in both left and right hand sides. It corresponds to the absence of a current in the  $z$  direction. The equations for the  $x$  and  $y$  components read

$$e\mathbf{E}_2 n_2 \rho(Z) = 2 \int \frac{d\varepsilon}{2\pi\hbar} \int \frac{d^2 p_2}{(2\pi\hbar)^2} \mathbf{p}_2 \exp \left\{ ie\hbar \mathbf{E}_2 \left[ \overleftarrow{\partial^\varepsilon} \overrightarrow{\partial_2^\mathbf{p}} - \overleftarrow{\partial_2^\mathbf{p}} \overrightarrow{\partial^\varepsilon} \right] \right\} I_z \quad (5)$$

where

$$n_2 \rho(Z) = -2i \int \frac{d^4 p}{(2\pi\hbar)^4} G^<(p, Z) \quad (6)$$

is the electron density, whose  $Z$  distribution is determined by a function  $\rho(Z)$  normalized to unity, meaning that  $n_2$  is the in-plane electron density;  $Z$  and  $\mathbf{p}$  are Wigner variables for the gauge invariant Green functions [9, 10];

$$I_z = \int \frac{dp_z}{2\pi\hbar} \exp \left\{ \frac{i}{2}\hbar \left( \overleftarrow{\partial^Z} \overrightarrow{\partial^{p_z}} - \overleftarrow{\partial^{p_z}} \overrightarrow{\partial^Z} \right) \right\} \times \\ \text{Tr} (\sigma_Z \Sigma(Z, p_z) \mathbf{G}(Z, p_z) - \mathbf{G}(Z, p_z) \Sigma(Z, p_z) \sigma_Z). \quad (7)$$

Here the Groenewold [14] notations are used, according which left and right arrows over the differential operators  $\partial^\xi \equiv \frac{\partial}{\partial \xi}$  denote the operators, which act either on the left or the right functions in the product, respectively.  $\mathbf{G}(Z, p_z)$  and  $\Sigma(Z, p_z)$  are matrices of the Keldysh Green functions and mass operators. To shorten the notations their dependencies on the variables  $\mathbf{p}_2$  and  $\varepsilon$  were suppressed.

The mass operator accounting for the  $s$  (in the  $xy$  plane) scattering by the potential (3) reads

$$\Sigma^{<>}(\varepsilon; Z, p_z) = \frac{\overline{U^2}}{(2\pi\hbar)^3} \int d^2p_2 dp'_z G^{<>}(\mathbf{p}_2, \varepsilon; Z, p'_z) \tilde{l}(p_z - p'_z). \quad (8)$$

where  $\tilde{l}(p_z)$  is the Fourier transform of the correlation function (4). Eq.(5) is solved in the linear, with respect to the driving electric field  $\mathbf{E}_2$ , approximation. Then the  $s$  scattering means that the mass operators (8) are at equilibrium.

It is important now to distinguish the  $Z$  and  $p_z$  dependencies of the quantities in Eq.(5). We consider the case when the Fermi level lies in the lowest subband of the confinement potential  $V(Z)$ , then at equilibrium

$$\rho(Z) = \bar{\rho}_0(Z) \equiv \frac{1}{u\sqrt{\pi}} \exp\left(-\frac{Z^2}{u^2}\right). \quad (9)$$

$$G^{(0)<>}(\mathbf{p}_2, \varepsilon; Z, p_z) = G_2^{(0)<>}(\mathbf{p}_2, \varepsilon) \rho_0(Z, p_z|u),$$

where  $G_2^{(0)<>}(\mathbf{p}_2, \varepsilon)$  is the equilibrium two dimensional Green functions, and

$$\rho_0(Z, p_z|u) = 4\pi\hbar u \sqrt{\pi} \exp\left\{-\frac{u^2 p_z^2}{\hbar^2}\right\} \bar{\rho}_0(Z) \quad (10)$$

(see discussion of the shape of the Wigner quasi-distribution function of a harmonic oscillator in [10]).

The time  $\tau_2$  of the electron scattering in the  $xy$  plane at equilibrium does not depend on the variables  $Z$  or  $\mathbf{p}_2$  and is defined as

$$\frac{\hbar l(0)}{\tau_2} \int \frac{dp'_z}{2\pi\hbar} \rho_0(Z, p'_z) \tilde{l}(p_z - p'_z) = i(\Sigma^{(0)<}(Z, p_z, \varepsilon) - \Sigma^{(0)>}(Z, p_z, \varepsilon)). \quad (11)$$

with  $l(0) = 1/(r_c\sqrt{2\pi})$ .

From the principle of the detailed balance for the quantum collision integral [8], we conclude that the nonequilibrium parts of the two above Green functions coincide, i.e.  $\delta G^>(Z, p_z; \mathbf{p}_2, \varepsilon) = \delta G^<(Z, p_z; \mathbf{p}_2, \varepsilon) \equiv \delta G(Z, p_z; \mathbf{p}_2, \varepsilon)$ . However, their  $Z$  and  $p_z$  dependencies do not necessarily coincide with those of the equilibrium Green functions (9). These dependencies can be found from the integral equation, which follows from Eq. (5). A Gaussian dependence of the nonequilibrium part of the Green function solves this equation. We look for a solution in the form

$$\delta G(\mathbf{p}_2, \varepsilon; Z, p_z) = \delta G_2(\mathbf{p}_2, \varepsilon) \rho_0(Z, p_z|b) \quad (12)$$

where  $\rho_0(Z, p_z|b)$  is described by Eq. (10), in which the length  $u$  is substituted for an unknown length  $b$ . Here  $\delta G_2(\mathbf{p}_2, \varepsilon)$  is the (yet unknown) nonequilibrium correction to the 2D Green functions, Eq. (9).

We now introduce all the above assumptions into the collision integral (7), and after some tedious but straightforward calculations arrive at the expression

$$I_z(Z) = -i \frac{\tilde{r}_c}{ub\tau_2} \delta G_2(\mathbf{p}_2, \varepsilon) \exp \left\{ -Z^2 \left( \frac{1}{\tilde{u}^2} - \frac{\tilde{r}_c^2}{2\tilde{u}^4} \right) \right\} \quad (13)$$

where

$$\frac{1}{\tilde{u}^2} = \frac{1}{u^2} + \frac{1}{b^2}$$

and

$$\frac{1}{\tilde{r}_c^2} = \frac{1}{r_c^2} + \frac{1}{u^2} + \frac{1}{b^2}. \quad (14)$$

As a result the kinetic equation (5) acquires the form

$$e\mathbf{E}_2 n_2 \bar{\rho}_0(Z) = 2 \int \frac{d\varepsilon}{2\pi\hbar} \int \frac{d^2 p_2}{(2\pi\hbar)^2} \mathbf{p}_2 I_z(Z). \quad (15)$$

Now we compare the  $Z$  dependence of the right hand side of the kinetic equation (15), determined by Eq. (13), and that of the left hand side, determined by the distribution  $\bar{\rho}_0(Z)$ . The requirement that these dependencies coincide, leads to the value

$$\frac{1}{b^2} = -\frac{1}{r_c^2} + \sqrt{\frac{1}{r_c^4} + \frac{1}{u^4}}, \quad (16)$$

which determines the scale of the  $Z$  dependence of the nonequilibrium Green function (12).  $b$  is now the effective width of the channel which does not necessarily coincide with the width  $u$  of the electron density profile. One can readily see that  $b \approx u$  only in the limit of the large correlation radius  $r_c \gg u$ . However, it may become very large,  $b \approx \frac{u^2}{r_c} \gg u$ , in the quantum limit,  $r_c \ll u$ .

Eq. (15) allows one to find directly the first conditional moment of the nonequilibrium correction to the 2D-Green function

$$\overline{\delta \mathbf{G}_2} \equiv \int \frac{d\varepsilon}{2\pi\hbar} \int \frac{d^2 p_2}{(2\pi\hbar)^2} \mathbf{p}_2 \delta G_2(\mathbf{p}_2, \varepsilon) = i \frac{en_2\tau_2}{2} \frac{b}{\tilde{r}_c} \mathbf{E}_2 \quad (17)$$

and, hence, the density of the electric current in the quasi-2DEG becomes (see Eq. (45) in [10])

$$\mathbf{J}(Z) = -2ie \int \frac{d\varepsilon}{2\pi\hbar} \int \frac{d^2 p_2}{(2\pi\hbar)^2} \frac{\mathbf{p}_2}{m} \delta G_2(\mathbf{p}_2, \varepsilon) \bar{\rho}_0(Z|b) = \sigma_2 \frac{1}{\sqrt{2\pi\tilde{r}_c}} \exp \left( -\frac{Z^2}{b^2} \right) \mathbf{E}_2. \quad (18)$$

where  $\sigma_2 = \frac{e^2 n_2 \tau_2}{m}$  is the 2D-Drude conductivity.

Integrating Eq.(18) over  $Z$  one gets the conductivity of the quasi-2DEG,  $\sigma = \sigma_2 \frac{b}{\sqrt{2}\tilde{r}_c}$ , which strongly depends on the ratio of the width of the channel  $u$  and the correlation length  $r_c$  of the scattering potential. The Drude formula is applicable, if only long range fluctuations are characteristic of the scattering potential, i.e.,  $r_c \gg u$ , then  $\frac{b}{\sqrt{2}\tilde{r}_c} \rightarrow 1$ , and  $\sigma = \sigma_2$ . However, in the quantum limit,  $r_c \ll u$ , when the scattering potential fluctuates in the range smaller than the width of the channel, a strong deviation from the Drude formula is expected. The effective width of the channel, where the current actually flows,  $b \approx \frac{u^2}{r_c^2} \gg u$ , may become much larger than the width  $u$  of the electron density profile. As a result, the conductivity may also become much larger  $\sigma = \sigma_2 \frac{u^2}{r_c^2} \gg \sigma_2$ .

### III. APPLICABILITY OF THE LINEAR RESPONSE APPROXIMATION

The above results are obtained within the linear response approximation. In order to probe the applicability limits of this approximation, one considers the  $Z$  profile of the drift velocity. It can be found using the conditional moment (17) (see Eq. (43) in [10])

$$\mathbf{v}_{drift} = \frac{1}{\bar{\rho}_0(Z)} \int \frac{d\varepsilon}{2\pi\hbar} \int \frac{d^2 p_2}{(2\pi\hbar)^2} \frac{\mathbf{p}_2}{m} \delta G_2(\mathbf{p}_2, \varepsilon). \quad (19)$$

This definition corresponds also to the conventional equation  $\mathbf{J}(Z) = en_0(Z)\mathbf{v}_{drift}(Z)$ , in which  $n_0(Z) = n_2 \bar{\rho}_0(Z)$  is the electron density at equilibrium.

One can readily see that differing  $Z$  dependencies of the current and the electron densities lead to a growth of the drift velocity with increasing  $Z$ . This fact indicates that at however small electric field  $E_2$ , there are large enough distances  $Z$ , at which conditions can be achieved, when the linear approximation in  $E_2$  is violated. The detailed analysis requires extremely cumbersome and lengthy calculations, which cannot be presented here. However, the most important conclusions can be made in a simpler way. We obtain a rough but sufficient (upper limit) criterion of the applicability of the linear approximation by requiring that the additional energy acquired by an electron due to the current flow does not exceed the temperature, i.e.  $v_{drift} p_F \ll k_B T$  where  $p_F$  is the Fermi momentum. Hence, the linear approximation is violated if  $Z > Z^*$  where

$$Z^{*2} = \frac{u^2 b^2}{b^2 - u^2} \ln \left| \frac{e\tau_2 p_F}{m} \frac{u}{\tilde{r}_c} \frac{E_2}{k_B T} \right|.$$

It is important to emphasize that although this condition is obtained for the harmonic potential well, considered here, its meaning is more general. For any potential well exists a value  $Z^*$  so that at larger distances the linear approximation does not hold. At  $Z > Z^*$  one has to consider a nonlinear problem involving contributions of various inelastic scattering mechanism (say, electron-phonon interaction). However, without making detailed calculations one can understand that in this region the scattering becomes much stronger and the current flow profile is actually much lower, than that obtained in the linear approximation, hence, its contribution to the total current can be neglected. When integrating over  $Z$  one may just cut the integration at  $|Z| = Z^*$  which provides reasonable estimates for the nonlinear corrections to the 2D conductivity,

$$\frac{\delta\sigma}{\sigma} = -2 \int_{Z^*/b}^{\infty} e^{-x^2} dx.$$

$Z^*$  increases with the decreasing electric field, and at  $Z^* \gg b$  these corrections are hardly observable.

#### IV. INFLUENCE OF AN IN-PLANE MAGNETIC FIELD

It is worthwhile to discuss here the influence of an in-plane magnetic field  $\mathbf{B}$  on the conductivity. According to the intuition based on the classical ideas about the electron motion, we do not expect any influence of an in-plane magnetic field, especially if it is directed *parallel* to the electric field. Nevertheless, as we demonstrate below, such an influence exists and a positive longitudinal magnetoresistivity is expected. We have to start again from Eq. (31) of our paper [10] and carry out the same procedure as described above. First, one can readily check that Eq. (5) retains its form in the case of  $\mathbf{B} \parallel \mathbf{E}_2$  and the magnetic field  $\mathbf{B}$  does not appear explicitly. However, it influences the shape of the equilibrium Wigner function [10], which becomes now

$$\rho_0(Z, p_z | u, \nu) = 4\pi\hbar\nu^{-2} \exp\left\{-\frac{u^2 p_z^2}{\hbar^2 \nu}\right\} \exp\left\{-\frac{1}{u^2 \nu^3} \left[z + \frac{p_x}{\hbar} \frac{u^2}{l_B^2}\right]^2\right\} \quad (20)$$

where

$$\nu = \left(1 + \frac{u^4}{l_B^4}\right)^{1/2}$$



and  $l_B^2 = \frac{\hbar c}{eB}$  is the magnetic length. We use this function in order to repeat the calculations carried above for the case of  $B = 0$ . The solution  $\delta G(\mathbf{p}_2, \varepsilon; Z, p_z)$  of the quantum kinetic equation will be again looked in the form (12) where the function (20) with  $b$  substituted for  $u$  is used instead of  $\rho_0(Z, p_z|b)$ .

Then we get an equation determining the parameter  $b$  as a function of the magnetic field

$$\frac{u^2}{b^2(\nu)} = \frac{\nu^3}{2\nu^3 - 1} \left\{ -\frac{\nu^4 - 1}{2\nu^3} - \frac{u^2}{r_c^2} + \sqrt{\left( \frac{\nu^4 - 1}{2\nu^3} + \frac{u^2}{r_c^2} \right)^2 + \frac{2\nu^3 - 1}{\nu^6}} \right\}$$

As a result the density of the magnetic field dependent electric current (18) in the quasi-2DEG becomes

$$\mathbf{J}(Z, \nu) = \sigma_2 \frac{1}{\sqrt{2\pi}\tilde{r}_c(\nu)} \exp\left(-\frac{Z^2}{b^2(\nu)}\right) \mathbf{E}_2. \quad (21)$$

where

$$\frac{u^2}{\tilde{r}_c^2(\nu)} = \frac{u^2}{r_c^2} + \frac{1}{2}(\nu + \nu^{-3}) \left(1 + \frac{u^2}{b^2}\right).$$

Integrating Eq.(21) over  $Z$  the magnetic field dependent conductivity of the quasi-2DEG is,

$$\sigma(\nu) = \sigma_2 \frac{b(\nu)}{\sqrt{2\pi}\tilde{r}_c(\nu)}$$

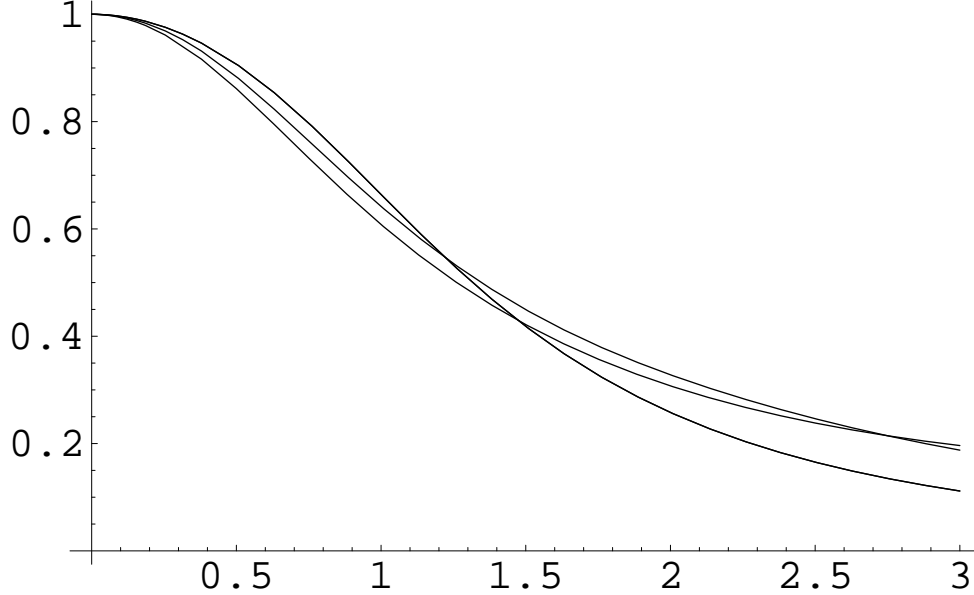


FIG. 1: Relative 2D resistivity as a function of the longitudinal magnetic field in units  $u^2/l_B^2$  for different ratios of the parameter  $u/r_c$  from 0 to 2. The curves nearly coincide.

Figure 1 shows the relative variation of the resistivity  $\rho(\nu)/\rho(0) = \sigma(0)/\sigma(\nu)$  as a function of the magnetic field in units  $u^2/l_B^2$  for four values of the parameter  $u/r_c$ : 0, 0.5, 1, 2. The curves for  $u/r_c = 0$  and 0.5 are indistinguishable in this plot. Two other curves for the higher values of the parameter  $u/r_c$  also nearly collapse. In the high magnetic field limit the resistivity rapidly decreases. It happens when the magnetic length  $l_B$  becomes much smaller than the width of the channel. Then the dynamics of electrons is controlled by the magnetic field and the confining potential well becomes actually irrelevant.

Experimentally this limit can hardly be achieved. If we take a typical width of the channel to be 100Å then the high magnetic field limit is reached only in the field measured in hundreds or even thousands of Tesla. It is much more realistic to expect some measurements carried out in the low field limit ( $l_B > u$ ) where a negative magnetoresistivity can be expected. We can write the low field expansion of the relative magnetoresistivity as

$$\frac{\rho(B)}{\rho(0)} = 1 - k \frac{u^2}{l_B^2}$$

where

$$k = \frac{2 + 7v^2 + 5v^4 + \sqrt{1 + v^4} - v^4(-4 + 5\sqrt{1 + v^2})}{4\sqrt{1 + v^4}(1 + \sqrt{1 + v^4})(-v^2 + \sqrt{1 + v^4})}$$

The dependence of this coefficient on the parameter  $v^2 = u^2/r_c^2$  is nearly linear as shown in Figure 2 and starts from  $k(0) = 3/8$  at  $v = 0$ . All the above data indicate that the resistivity may increase by a factor of 2 if the magnetic length  $l_B$  becomes comparable with the width of the channel  $u$ .

A positive longitudinal magnetoresistivity was observed in SiGe layers [15, 16]. The authors attributed the effect to the electron spin polarization (a theory is presented in [7]). This explanation seems to be quite reasonable since the electron concentrations in these experiments are so low that a complete spin polarization is possible. The mechanism proposed in this paper may become more important at higher electron concentration. We do not know currently about any relevant experimental data and the question remains open.

The results of this paper emphasize that the difference between the 2DEG and quasi-2DEG, however subtle it is, may be of an utter importance. If the scattering potential comprises fluctuations with a small enough correlation radius, the profile of the electron current may become broader than the profile of the electron density. A longitudinal negative

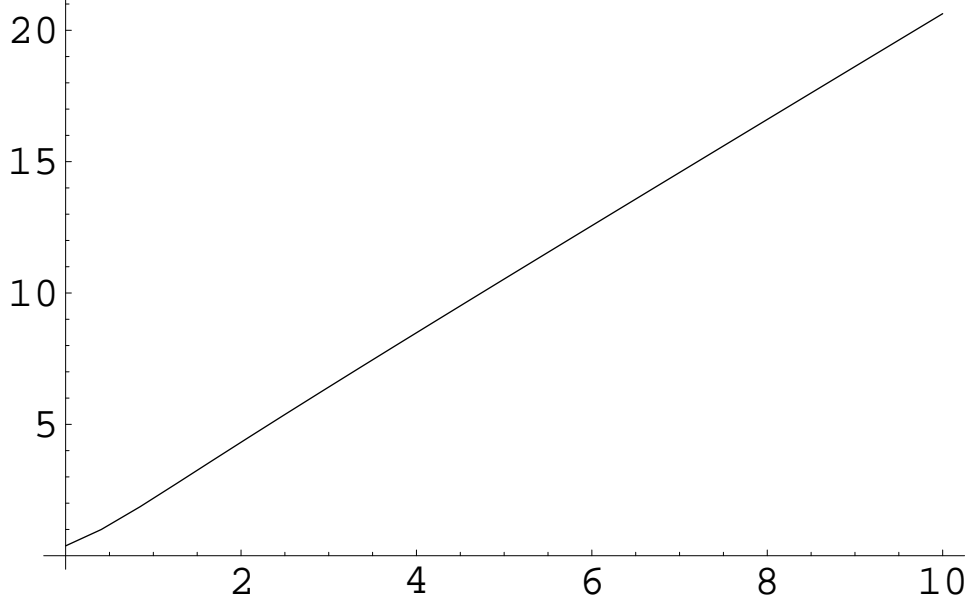


FIG. 2: The dependence of the coefficient  $k$  in the low field expansion of the magnetoresistivity on the parameter  $v^2$ .

magnetoresistivity is proposed as an experimentally observable consequence.

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